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PASSIVE RANGING AND TARGET MOTION ANALYSIS

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13. ABSTRACT

A review is made of various methods for passive ranging and target motion analysis, including some new methods based on three tracking legs. Estimators for range and other parameters are compared with respect to their responses to errors from four major sources. A program for additional tests and comparisons is outlined.

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PASSIVE RANGING AND TARGET MOTION ANALYSIS*

Introduction

Passive ranging and target motion analysis have been studied for many years. A few of the many methods that have been developed have been adopted for use. The methods in use have not been satisfactory in all respects, and the search continues.

There appears to be no "best" way to proceed. That is, there is no method that is superior to all other methods in all respects. Under these conditions the advocates of various methods argue their cases by the adversary procedure. A better procedure would be to examine the major characteristics of the leading contenders on an objective basis, with the same tests and conditions applied to all. The main question remaining then would be that of the characteristics that are important in a particular application, and the relative weights that should be assigned to the characteristics in choosing an estimator for the range or for a complete set of target motion parameters.

The purpose of this report is to describe some of the leading contenders, to discuss their characteristics to the extent that they are known, and to outline a test and comparison program.

Approach

The major characteristics of a range estimator, or an estimator of some other parameter, are the following:

(a) Formula accuracy
(b) Sensitivity to error measurements
(c) Ruggedness to target maneuvers

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All three characteristics are important in most applications, but the relative weights may be different.

By formula accuracy we mean the accuracy of the estimator when there are no measurement errors of any kind and the target motion is strictly linear. The reader who has a classroom knowledge of statistics and little experience with estimation problems of the type we are considering may be surprised that we would consider any estimators that do not have formula accuracy. Yet, as will be seen later, some estimators of this type are not only useful but may be the best we can do in regard to certain characteristics.

The above remarks are made to dispel any doubt that the estimation of range and range rate from bearings only is a difficult problem. The theory of unbiased estimators having minimum variance just doesn't apply. All estimators of range and range rate that we know of are biased. Some are biased through formula inaccuracies, and all are biased stochastically.

Almost all estimators are derived under the assumption of linear target motion. This assumption in the derivation of possible estimators is justified, since bearing accuracies are not sufficient to predict non-linear target motion. That is, the best that we can do with present accuracies is to predict the linear component of target motion, and even this much is in doubt, in my opinion. But, whatever assumptions are made in the derivation, we must recognize the fact that the target may, and often does, have motion that is far from linear. Hence, the estimator, whether derived from the linear assumption or not, should be subjected to tests against non-linear motion.

When we consider the difficulties of the problem and the several likely sources of large biases, it seems evident that highly sophisticated techniques are not justified. At least they should not be used until it can be demonstrated that they significantly

reduce the total bias. The elimination of a random component of the error in the range estimate, for example, will do little to increase the effectiveness of the estimate, if the standard deviation of the random component is small relative to the total bias. A significant improvement in the estimate requires a significant reduction in the dominant error, in general. And, the dominant error usually is a bias.

There are four sources of bias, as follows:

(1) Inaccuracies in the formulas

(2) Biases in the bearing observations, or other observations

(3) Stochastic bias

(4) Non-linear target motion

The first one has been discussed above. Biases in the bearing observations are known to exist. The bias depends on the angle off the bow of the tracking submarine, and usually is called the delta bias. The third and fourth sources of bias also have been mentioned above. We believe that all four sources of bias must be included in an adequate analysis.

If the target maneuvers radically during the tracking interval, the fourth bias almost certainly is the dominant bias. If the maneuver is mild, one of the other biases may be the dominant one.

The net effect of the biases and random errors will depend on the use that is made of the estimates, and the measure of performance that is chosen. In general, a reduction in the magnitude of the dominant bias component will produce a net gain, while a reduction in the dominant variance may produce a net gain or a net loss. To avoid considering various applications and measures of performance we will concentrate on the bias components, for which the favorable direction is known.

For some purposes a range estimate is sufficient. For other purposes a complete TMA is required. The parameter estimators in a complete TMA may include a range estimator directly, or require an additional calculation. Complete TMA estimators and range-only estimators are included in this preliminary comparison of bias components.

Notation

We use the standard naval coordinate system. For the ith observation let

t_i = time

b_i = bearing of the target from the tracking submarine

r_i = range of the target from the tracking submarine

u_{ri} = range component of target velocity measured outward

u_{bi} = normal component of target velocity measured clockwise

v_{ri} = range component of own-submarine velocity measured outward

v_{bi} = normal component of own-submarine velocity measured clockwise

It will be convenient to use

B_i = bearing observation, or estimate of b_i

R_i = estimate of r_i

$t_{ij} = t_j - t_i$, $B_{ij} = B_j - B_i$, $S_{ij} = \sin B_{ij}$, $C_{ij} = \cos B_{ij}$

Estimates From Two Tracking Legs

We assume that the tracking submarine uses two essentially linear tracking legs on two quite different courses, of the type described as "Lag-lead", for example. Four bearing observations are sufficient for a complete TMA, on the assumption of linear target motion. The corresponding estimator for the range r_4 at the time t_4 of the fourth observation is

$$R_4^{(1)} = \frac{t_{14}t_{24}t_{34}(v_{b1}S_{23}+v_{b2}S_{31}+v_{b3}S_{12})}{t_{12}t_{34}S_{13}S_{24} - t_{13}t_{24}S_{12}S_{34}} \qquad (1)$$

This equation is equivalent to equation (16) of reference (a). It is the only exact estimator for r_4 from four bearing observations with the linear assumption. Estimators for other parameters sufficient for a complete TMA, also can be written.

Some inexact estimators for r_4 are the following:

$$R_4^{(2)} = (v_{b3} - v_{b1}C_{13} - v_{r4}t_{24}\beta_{12})/(\beta_{12} - \beta_{34}) \qquad (2)$$

$$R_4^{(3)} = (v_{b3} - v_{b1}C_{13})/(\beta_{12}' - \beta_{34}) \qquad (3)$$

where

$$\beta_{12}' = (S_{12}/t_{12})(C_{13}/C_{24}), \ \beta_{34} = S_{34}/t_{34} \qquad (4)$$

Equation (2) is equivalent to equation (6) of reference (a), and is obtained by assuming that the u_r component of target motion during the interval (t_1,t_4) is negligible. Equation (3) is equivalent to equation (2-23) of reference (b), and is obtained from equation (2) by assuming that the v_{r4} component of own-ship motion during (t_2,t_4) also is negligible.

A form of the Ekelund range estimator is obtained from equation (3) by letting t_2 approach t_1, t_4 approach t_3, and approximating C_{13} and C_{24} by 1.0.

The estimator is

$$R_4^{(4)} = (v_{b3} - v_{b1})/(\dot{B}_1 - \dot{B}_3) \qquad (5)$$

where $\dot{B}_1$ and $\dot{B}_3$ are estimates of the bearing rates at t_1 and t_3. This estimator usually is a better estimator for r_3 than for r_4. A slightly better estimator for r_4 is

$$R_4^{(5)} = (v_{b4} - v_{b2})/(\dot{B}_2 - \dot{B}_4) \qquad (6)$$

which can be derived directly from the assumption that the u_r and v_r components are negligible; see reference (c).

Other forms similar to those in equations (2), (3), (5) and (6) also have been used. In particular, the special form of equation (3) when $t_2 = t_3$ is called the DEVGROUP range estimate in reference (b).

A complete TMA can be obtained in several ways. First, we can use the exact solution based on four bearings. The range component of this solution is obtained from equation (1), and estimators for other parameters can be written. However, we do not recommend it. The range estimator $R_4^{(1)}$ is very sensitive to random bearing errors and to mild deviations of the target from a linear course; and the estimator for u_{r4} is even more so. The difficulty stems from the fact that the denominator in (1) is a difference of two terms, which, for a standard approach, have the same sign and order of magnitude. Often the magnitude of the difference between the terms is small relative to the magnitudes of the terms. Hence, a small fractional error in a term produces a large fractional error in the difference.

A simple TMA based on four bearing estimates is the following:

<u>TMA No. 1</u>

B_4 = estimate of bearing at $t = t_4$

R_4 = estimate of range at $t = t_4$, from equation (2), (3), or (5)

U_{r4} = estimate of $u_{r4} = 0$

$$U_{b4} = \text{estimate of } u_{b4} = v_{b4} + R_4 \, S_{34}/t_{34} \qquad (7)$$

The four estimates above give us the (polar-coordinates) position and velocity components at $t = t_4$. The use of 0 for U_{r4} is consistent with the assumptions made in obtaining the range estimators. An exact estimator for U_{b4} is

$$U_{b4} = v_{b4} + R_3 \, S_{34}/t_{34}$$

Hence, the estimators $R_4^{(2)}$, $R_4^{(3)}$, and $R_4^{(4)}$, which are better estimators for r_3 than for r_4, are preferred over the estimator $R_4^{(5)}$ in the estimator (7) for U_{b4}.

Accurate estimates of the bearing rates that appear in the Ekelund range estimator can't be obtained from the four (B_i, t_i) pairs alone. We need frequent bearing observations. Then the bearing rates can be estimated from the smoothed bearing-time plot, or computed directly from the bearing observations by using some method of smoothing. A method using exponential smoothing in reference (c) was found to be satisfactory. If a more accurate estimate of the bearing rate can be obtained from the bearing-time plot at a time that is not the end of a tracking leg, t_2 and t_4 at which the velocities and bearing rates are estimated in equation (6) can be replaced by earlier times. The increase in accuracy of the bearing rates may more than offset the lag in the range estimate.

If we take many bearing observations on the two tracking legs, other methods of target motion analysis, such as CHURN, can be used. Also, various filtering and smoothing techniques can be applied. These methods and techniques will be discussed briefly.

Discussion of Range Estimators From Two Tracking Legs

The range estimator $R_4^{(1)}$ has no bias from formula accuracy, which is the cnly advantage it has over the other range estimators. The biases from formula inaccuracies in the other range estimators vary from trivial to serious, depending on many factors, such as the range at the start of the tracking interval, the duration of the tracking interval, and the motions of the target and own submarine during tracking. Biases exceeding 15 percent are common, and biases exceeding 30 percent are not unusual. Some results from computer simulations are given in references (b) and (c).

The bias in the bearing observations usually is called the delta bias. It depends on the angle off the bow of the tracking submarine at which the bearing is taken, and it usually varies slowly with changes in this angle. Hence, the delta bias is nearly constant for a tracking leg, but varies from leg to leg. Hence, by taking B_1 and B_2 on the first leg, and B_3 and B_4 on the second leg, the errors in S_{12}, S_{34} and the bearing rates can be kept small. Thus, $R_4^{(2)}$, $R_4^{(3)}$, $R_4^{(4)}$, and $R_4^{(5)}$ have a distinct advantage over $R_4^{(1)}$ for this bias component. A partial analysis for $R_4^{(5)}$ was made in reference (c).

Random bearing errors also produce larger errors in $R_4^{(1)}$ than in the inexact estimators, both in bias and in variance. The biases arise from the fact that the range estimators are not linear in the bearing inputs. This source of bias in the range estimator probably is not the dominant bias for the range estimators displayed above, although we do not know of any results that apply to the question. (However, it may be a serious bias in the CHURN TMA under some circumstances; see the discussion of CHURN in a later section.)

If the target departs mildly from a linear course, say 10 degrees change in course near the middle of the tracking interval, the bias in $R_4^{(1)}$ is larger than that in the other estimators. If the target maneuvers radically, say in the middle half of the tracking interval, all range estimators are useless. To avoid accepting estimates that are grossly in error, we need tests that will reject most of the solutions that are obtained when the target maneuvers radically, and hopefully not reject too many good solutions. Tests of this kind, using the Ekelund range estimator for the last two tracking legs and a three-bearing-rate range estimator (equation (23) following) for the last three tracking legs, have been developed and tested in reference (c). These tests will be discussed briefly after the derivations for three tracking legs are made.

Estimates From Three Tracking Legs

Assume that three tracking legs are used and we obtain two bearing estimates, or observations, on each leg. From these six bearings we can derive an exact range estimator that has the desirable characteristics of the inexact estimators $R_4^{(2)}$ and $R_4^{(3)}$ in equations (2) and (3). That is, by using six bearings, instead of four, we can eliminate the biases in $R_4^{(2)}$ and $R_4^{(3)}$ from formula inaccuracies, without the necessity of using a sensitive estimator of type $R_4^{(1)}$ in equation (1).

We will use special cases of three general equations, as follows:

$$r_j C_{jk} = r_i C_{ik} + (u_{rk} - v_{rk}) t_{ij} \tag{8}$$

$$u_{bk} = v_{bk} + r_j S_{jk}/t_{jk} \tag{9}$$

$$u_{bj} = u_{bk} C_{jk} + u_{rk} S_{jk} \tag{10}$$

We note that we can replace the letter u by the letter v in equation (10).

Assume that we wish to find estimators for r_6 and u_{r6}. First, we use the second leg with indices 3 and 4 and the third leg with indices 5 and 6. Special cases of equations (8), (9), and (10) are

$$r_6 = r_5 C_{56} + (u_{r6} - v_{r6}) t_{56} \tag{11}$$

$$r_6 = r_3 C_{36} + (u_{r6} - v_{r6}) t_{36} \tag{12}$$

$$u_{b6} = v_{b6} + r_5 S_{56}/ t_{56} \tag{13}$$

$$u_{b4} = v_{b4} + r_3 S_{34}/ t_{34} \tag{14}$$

$$u_{b4} = u_{b6} C_{46} + u_{r6} S_{46} \tag{15}$$

When we eliminate u_{b4}, u_{b6}, r_3, and r_5 from these five equations we obtain the equation

$$G_1 r_6 + H_1 u_{r6} = M_1 \tag{16}$$

where

$$G_1 = \beta_{56}\, C_{36}\, C_{46}/C_{56} - \beta_{34},\quad H_1 = S_{45}\, C_{36}/C_{56} + \beta_{34}\, t_{36} \tag{17}$$

$$M_1 = v_{b4}\, C_{36} - v_{b5}\, C_{36}\, C_{46}/C_{56} + v_{r6}\, t_{36}\, \beta_{34} \tag{18}$$

$$\beta_{jk} = S_{jk}/t_{jk} \tag{19}$$

If we replace the subscript 3 by 1 and 4 by 2, equations (16), (17), and (18) become

$$G_2\, r_6 + H_2\, u_{r6} = M_2 \tag{20}$$

$$G_2 = \beta_{56}\, C_{16}\, C_{26}/C_{56} - \beta_{12},\quad H_2 = S_{25}\, C_{16}/C_{56} + \beta_{12}\, t_{16} \tag{21}$$

$$M_2 = v_{b2}\, C_{16} - v_{b5}\, C_{16}\, C_{26}/C_{56} + v_{r6}\, t_{16}\, \beta_{12} \tag{22}$$

where, now, $\beta_{12} = S_{12}/t_{12}$.

We now can solve equations (16) and (20) simultaneously for r_6 and u_{r6}. Using capital letters to indicate that the bearings and own-ship velocity components are subject to errors, the estimators for r_6 and u_{r6} are

$$R_6 = (M_1\, H_2 - M_2\, H_1)/(G_1\, H_2 - G_2\, H_1) \tag{23}$$

$$U_{r6} = (G_1\, M_2 - G_2\, M_1)/(G_1\, H_2 - G_2\, H_1) \tag{24}$$

The estimator R_6 in equation (23) has formula accuracy, but is much less sensitive to errors than is the exact estimator $R_4^{(1)}$ in equation (1).

Now let t_1 approach t_2, t_3 approach t_4, and t_5 approach t_6. Then the G, H, M factors in our estimators (23) and (24) become

$$\left.\begin{aligned} &G_1 = \dot{B}_6\, C_{46}^2 - \dot{B}_4,\quad H_1 = S_{46}\, C_{46} + \dot{B}_4\, t_{46}, \\ &M_1 = v_{b4}\, C_{46} - v_{b6}\, C_{46}^2 + v_{r6}\, t_{46}\, \dot{B}_4 \end{aligned}\right\} \tag{25}$$

$$\left.\begin{array}{l} G_2 = \dot{B}_6 C_{26}^2 - \dot{B}_2, \; H_2 = S_{26} C_{26} + \dot{B}_2 t_{26}, \\ M_2 = v_{b2} C_{26} - v_{b6} C_{26}^2 + v_{r6} t_{26} \dot{B}_2 \end{array}\right\} \tag{26}$$

where $\dot{B}_2$, $\dot{B}_4$, $\dot{B}_6$ are estimates of the bearing rates at times t_2, t_4, t_6.

Equations (23)(24)(25)(26) are equivalent to equations (A-30)(A-31)(A-32) of reference (c), where the three-bearing-rate form was derived directly, not via the six-bearing form. If it is easier to estimate bearing rates and and own-ship velocity components at times $t_{2'}$, $t_{4'}$, $t_{6'}$ that are slightly less than the times t_2, t_4, t_6 at the ends of the tracking legs, the corresponding estimators (23) and (24) apply to time $t_{6'}$. They will have a slight bias of the first kind when used as estimators of r_6 and u_{r6}. If desired, the bias in the range estimator could be removed by using an equation of type (8), namely,

$$R_6 = (1/C_{66'}) \; [R_{6'} + (U_{r6'} - v_{r6'}) \; t_{6'6}] \tag{27}$$

but the gain in formula accuracy probably is outweighed by the error introduced by the use of $U_{r6'}$. The estimator $R_{6'}$ is recommended, instead of R_6 from equation (27), when it is desirable to compute the bearing rate on the last leg at a time $t_{6'}$ other than t_6.

We can use the estimators (23) and (24) to get a complete set of estimators for a TMA. Some TMA methods and the corresponding estimators are as follows:

<u>TMA No. 2</u>

B_6

R_6 from equation (23), using the six-bearing forms for the G, H, M factors

$U_{r6} = 0$

$U_{b6} = v_{b6} + R_6 \, S_{56}/t_{56}$

TMA No. 3

B_6

R_6 from equation (23), using the three-bearing-rate forms for the G, H, M factors

$U_{r6} = 0$

$U_{b6} = v_{b6} + R_6 \dot{B}_6$

TMA No. 4

Same as TMA No. 2, except that U_{r6} is obtained from equation (24), using the six-bearing forms.

TMA No. 5

Same as TMA No. 3, except that U_{r6} is obtained from equation (24), using the three-bearing-rate forms

In TMA No. 2 and TMA No. 4 a slightly better estimator of u_{b6} is obtained by using an equation of type (8) to obtain an estimator for r_5 to replace R_6 in the equation for U_{b6}, since

$$u_{b6} = v_{b6} + r_5 \, S_{56}/t_{56}$$

However, the gain is slight and doesn't justify the additional complication.

TMA No. 3 and TMA No. 5 have been studied in reference (c), and have been compared with respect to their effectiveness in corrected-intercept control of the Mk48 torpedo. There is little difference in effectiveness, and that is in favor of TMA No. 3 for the assumed error parameters.

Discussion of Estimators From Three Tracking Legs

The estimators R_6 and U_{r6} in equations (23) and (24) are exact and are less sensitive to bearing errors than the corresponding

exact estimators from two legs. The sensitivity properties of R_6 are close to those of the inexact range estimators for two legs. When alternate lag- and lead-legs are used for tracking, the value of $|G_1 H_2|$ usually is an order of magnitude greater than $|G_2 H_1|$, thereby eliminating a major source of sensitivity. Some results for R_6 with the three-bearing-rate forms of G, H, M are given in reference (c).

The problem of estimating u_{r6} from bearing observations alone is a difficult one. After a careful study of the problem we have come to the conclusion that the estimator

$$U_{r6} = 0 \tag{28}$$

is hard to beat. By accepting a 100 percent bias we avoid all other biases, some of which can easily be greater than 100 percent. We also have zero variance. The estimator (24) is quite sensitive to bearing errors and to non-linear target motion, although it is less sensitive to these variations than other estimators, such as CHURN, that we have studied.

In some applications the range component of target velocity is not needed, or it plays a minor role. For example, the lead angle for interception of the target, assumed to be in linear motion, doesn't require the range component of target velocity, if the guide point is on the current bearing line. Also, any realizable value of the range component of target velocity has little effect on the lead angle when the guide point is close to the current bearing line. Hence, under these conditions any estimator that doesn't yield impossible values can be used. And impossible values can be avoided when using the estimator (24) by a simple acceptance/rejection test based on the magnitude of the estimate. A simpler way to avoid impossible values is to use the estimator (28), which proved to be slightly superior to the estimator (24) with an acceptance/rejection test in some fire-control

problems, as reported in reference (c). This conclusion does not necessarily apply to other uses of the u_r estimators.

Acceptance/Rejection Tests

The use of three tracking legs requires a tracking interval approximately 50 percent larger than that required for two tracking legs, and provides some advantages. First, it enables us to obtain a comparatively rugged range estimator (23) that has formula accuracy. Second, it provides an estimator (24) of the range component of velocity that might be competitive with the simple estimator (28) in some applications. Third, it provides us with a means of constructing simple acceptance/rejection tests to avoid gross errors produced by non-linear target motion. In our opinion, it is only the third advantage that justifies the use of more than two tracking legs.

Some acceptance/rejection tests are described in reference (c), together with the results obtained from a computer simulation. Three tracking legs are run and the test is made. If the outcome is positive, the solution is accepted. If the outcome is negative, the solution is rejected and another tracking leg is run, continuing until the first positive outcome. A simple test of this type for the range estimate is the following:

Let

R_E = Ekelund range estimate from the last two tracking legs, using an estimator of type (6) in which the v_b and $\dot{B}$ components are estimated for the end of the tracking legs

R_D = range estimate from the last three tracking legs, using an estimator of type (23) with the G, H, M factors computed from the bearing rates at the ends of the last three legs

(r_{min}, r_{max}) = acceptable range interval

a_r = coincidence factor, $0 < a_r < 1$

The test conditions are:

$$a_r < R_E/R_D < 1/a_r \tag{29}$$

$$r_{min} < R_E < r_{max}, \; r_{min} < R_D < r_{max} \tag{30}$$

If all conditions are satisfied, accept the solution; otherwise, reject it. Then there remains the question of which range estimate, R_E or R_D, to use when the solution is accepted. This question was made a part of the simulation study in reference (c), as was the question of what values to use for a_r, r_{min}, and r_{max}.

At first, only condition (29) was used. Conditions (30) were added to eliminate a few very wrong solutions for which condition (29) was satisfied. For this purpose, it was found that the values for r_{min} and r_{max} could be chosen to be any reasonable limits within which the range could lie and the desired (fire-control) objectives could be achieved. The bias in R_E from formula inaccuracy made it necessary to use what might appear to be small values of a_r, to avoid rejecting too many "good" solutions.

Other acceptance/rejection tests were tried, including one that seldom accepts a poor solution. However, it requires four tracking legs and often rejects good solutions. A simple test of the form

$$|U_r| < u_{max} \tag{31}$$

was used when the estimator (24) was used for the range component of velocity.

Time Correction

This method of range estimation is described in reference (b). A critique of the method was made in reference (a), and a reply was made in reference (d). We believe that a fair and impartial summary of the method is as follows:

Time correction is a method of obtaining an estimator R^* for the range at a time t^* that is computed from the bearing observations and the corresponding times at which the observations are made. The equation for R^* contains two additional bearings $\overline{B}$ and B^* that must be estimated by interpolation or extrapolation from the bearing-time plot; their use can be avoided by using small-angle approximations, the validity of which can't be stated without estimating the two bearings. The time t^* at which the estimator R^* applies is only partially under the control of the tracking submarine; it is unlikely to be close to the time at which a range estimate is needed.

In reply to the question of how one would get an estimate of the range at a time different from t^*, it is stated in reference (d) that a second time-corrected range estimate would be made and used to get a complete TMA, from which the desired range estimate presumably would be obtained. And we are referred to Chapter 5 of reference (b) for the details.

A method of getting a complete TMA from four bearings, taken on three tracking legs, is described in Sections 5.4 and 5.5 of reference (b). In our notation, the epoch t_1 occurs on the first leg, t_2 and t_3 on the second leg, and t_3 on the third leg, according to Figure 5-4. In our opinion, this method has several undesirable properties. Since t_1 and t_2 occur on different tracking legs, S_{12} will have a large bias from delta biases in the bearings; similarly, for S_{34}. These biases can be reduced significantly by pairing bearings on the same tracking leg, as explained earlier. Secondly, the four-bearing TMA obtained by

the method described in reference (b) is equivalent to (that is, identical to, in the absence of errors) the ordinary four-bearing TMA, as stated on page 5.8 of reference (b). Hence, it is very sensitive to random bearing errors and to small deviations from linear target motion. If we are willing to accept these sensitivities, why not use the simpler, and equivalent, direct solution, for which $R_4^{(1)}$ in equation (1) is the range estimator?

We believe that a better method, if it is desired to use time correction, is the following: Take two bearing observations on each tracking leg and pair them to avoid large biases from delta biases. Use legs 2 and 3 to get the estimator R_1^* at time t_1^*, and legs 1 and 3 to get the estimator R_2^* at time t_2^*. To get estimators for r_6 and u_{r6}, for example, use an equation of type (8) with $t_i = t_1^*$, $t_j = t_2^*$, $t_k = t_6$ and solve for u_{r6}. Then use another equation of type (8) with $t_i = t_1^*$, $t_j = t_k = t_6$ and solve for r_6.

We submit that the above TMA method, in the absence of errors, is identical with the direct method that resulted in the estimators R_6 and U_{r6} in equations (23) and (24). The time-correction method has all the errors of the direct method, plus a few more. The additional errors are those that occur in estimating the four bearings $\bar{B}_1$, B_1^*, $\bar{B}_2$, and B_2^* at the computed times $\bar{t}_1$, t_1^*, $\bar{t}_2$, and t_2^* from the bearing-time plot by interpolation or extrapolation; or alternatively, the biases that occur from the use of the small-angle approximations. Is there any gain that offsets these errors and the additional complications? We believe that the direct method is preferable, and that the very simple estimator (28) that is used in TMA No. 2 and TMA No. 3 may be even better for some purposes than the estimator (24).

The CHURN TMA

The CHURN TMA uses estimators for the rectangular coordinates (x_o, y_o) of the relative position of the target at the start of

tracking and for the rectangular components (u_x, u_y) of target motion. The estimators are obtained by minimizing a sum of (perhaps weighted) squares of residuals measured normal to the bearing lines.

The results of a study of the biases in the CHURN TMA are given and discussed in reference (c). Delta biases in the bearing errors produce large errors in the values of the position and motion parameters. Also, random bearing errors produce very large biases in the parameters under some conditions, which was not expected. Large deviations, and even moderate deviations, from a linear course produce large biases in the parameters.

The stochastic bias and variance were studied by Monte Carlo simulation. The very large biases from strictly linear motion and random bearing errors were so unexpected that they were ascribed to undetected errors in the computational procedure until they were understood and could be explained. When the mechanism that produces the biases was understood, a simple algorithm could be constructed to compute the stochastic bias without recourse to Monte Carlo simulation.

The stochastic bias occurs because a radial-coordinate residual is used in a rectangular coordinate system, thereby producing estimators for the parameters that are far from linear in the bearings. The sum of squares of the residuals can be written as a sum of two terms, one arising from biases in the solution and the other from the random variations in the bearings. The minimization of the sum of the two terms produces a biased solution that has a much smaller value than the unbiased solution for the random component of the sum and a non-zero value for the bias component of the sum. The bias in the solution is in the direction of predicting that the target track over the tracking interval lies closer to the tracking submarine than does the actual target track.

The magnitude of the stochastic bias depends on several factors, of which the duration of the tracking interval and the standard deviation in the bearing errors appear to be the most important. For tracking intervals of 10 minutes or less and standard deviations of 0.5 degrees or more, the stochastic bias is so large as to make the solution of doubtful value.

Large deviations from a linear course, initiated near the middle of the tracking interval, produce large errors in the parameters. In general, CHURN is slow to respond to changes in target course or speed after tracking has been in progress for a long period. Thus, changes in course or speed that occur late in the tracking interval will have little effect on the solution. This characteristic of CHURN is an important one in fire-control applications.

A change in target course or speed late in the tracking interval often produces a large bias in the lead angle. The estimate of range may not be greatly in error, but the estimate of the velocity component u_b normal to the bearing line, from which the lead angle is computed, will not reflect the current target motion.

The large lag in the lead angle from CHURN is intolerable in post-launch control of a wire-guided torpedo in the corrected-intercept mode. The sole purpose of post-launch tracking and control is the detection of changes in target motion that occur after launch, and in time to permit corrections to the torpedo course to be computed and applied. For this purpose we believe that TMA No. 3 and TMA No. 5--and, we prefer TMA No. 3--are superior to CHURN by a wide margin. If a moderately good estimate of the bearing rate for the current target motion can be obtained from the bearing-time plot or by the exponential-smoothing method used in reference (c), the estimate of the normal velocity component will be accurate enough to yield a good value for the lead angle.

Filtering and Smoothing Techniques

The purpose of filtering and pre-smoothing techniques is to remove the random bearing errors, to the extent that this is possible. For the sake of argument, assume that the random bearing errors can be reduced by several orders of magnitude through the use of a sophisticated technique, such as Kalman Filtering, and effectively eliminated thereby. What effects will this change have on the total error structure of the estimators?

Only the stochastic bias and the variance are affected. The biases from formula inaccuracies, from biases in the bearing errors, and from non-linear target motion are unaffected. The net effect on the total bias may be large or insignificant, depending on the magnitudes of the other biases relative to that of the stochastic bias. Only when the stochastic bias is the dominant bias will the net effect on the total bias be large. This dominance will occur in some cases with the estimators obtained from CHURN, particularly when the target motion is nearly linear and the tracking interval is short. We believe that the CHURN TMA is a likely candidate for the application of filtering techniques.

A large reduction in the random bearing errors may reduce the variances in the estimators from this source to the point that variations in other estimates, such as own-ship velocity components, are dominant. The net effect of a large reduction in the variance of an estimator depends on the application and the magnitude of the total bias. In fire-control applications the optimal variance usually is not zero when the total bias is large in magnitude, in which case a large reduction in the estimator variance may produce a net loss.

Another possible gain from the use of filtering techniques is that of obtaining more accurate estimates of bearing rates than

can be obtained from a smoothed bearing-time plot or by exponential smoothing, for use in our TMA No. 3 or TMA No. 5. The need for accurate estimates of bearing rate is particularly great in the corrected-intercept control mode that we proposed in reference (e) for use with degraded tracking. This use of filtering techniques may prove to be more important than other uses that have been proposed.

<u>Test and Comparison Program</u>

Some methods of computing biases and variances of the estimators are described in reference (c). We propose that these methods be reviewed, revised as necessary, and applied to the principal estimators and tests described above and to others that appear to be competitive.

Computer simulation has been found to be the simplest method of computing biases from non-linear target motion. This procedure also can be used to compute estimator biases from formula inaccuracies and from bearing biases, although simple analysis also can be used. The point here is that the use of computer simulation for the computation of biases from all three sources can be done with little more effort than that needed for the non-linear motion.

Biases and variances from random bearing errors can be handled by simple analysis in many cases. We had to resort to Monte Carlo simulation in the use of CHURN, and it may be necessary for some other estimators.

The analysis of errors should be extended to include errors in other parameters, such as own-ship position and velocity components. The analysis also should include tests of the accept/reject type.

References:

(a) ADL Technical Report ADL-72580-2, "Time Correction in Passive Ranging: Breakthrough or Bootstrap?", Contract No. N00014-70-C-0322, October 1970, Unclassified.

(b) COMSUBDEVGROUP TWO Report No. 2-69, Passive Ranging Manual (U), Volume III THEORY, Dec. 1969 Confidential.

(c) ADL Report NUWRES #12, "Control Modes and Acquisition Probabilities for Torpedo MK48 (U)", Contract No. N00140-68-C-0278, Jan. 1970, Final Report Confidential, Technical Appendices (bound separately) Unclassified.

(d) DHWA Memorandum, "Remarks on the Arthur D. Little Review of Time Correction (U)", Nov. 30, 1970, Confidential.

(e) ADL Technical Report ADL-72580-1, "Corrected-Intercept Control of Torpedo MK48 With Centroid Tracking", Contract No. N00014-70-C-0322, September 1970, Unclassified.